HEAT TRANSFER IN CONDENSATION ON A GROOVED SURFACE

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The process of condensation on a grooved surface is analyzed and the basic parameters of the process determined. Expressions are obtained for the heat-transfer coefficient.

The use of grooved surfaces in evaporation and condensation equipment to intensify heat exchange has increased in recent years. Among such equipment, we find threaded arterial heat pipes, heat pipes with longitudinal grooves, and others. When condensers are constructed from such heat pipes information on the heat-transfer coefficients for condensation on the grooved surface is necessary. General features of a model of the condensation process on a grooved surface were formulated in [1] for heat pipes. However, the method of analysis used therein proves very cumbersome, and does not permit clarification of the characteristic peculiarities of the process analyzed.

Below we will describe a more general model of the process of condensation on a grooved surface, the analysis of which will determine the basic parameters of the process, defining condensation, and expressions will be obtained for the heat-transfer coefficient.

In low-temperature heat pipes the thermal conductivity of the construction material is, as a rule, several hundred times higher than that of the heat-exchange agent. Therefore, the main thermal resistance in film condensation is connected with the liquid film formed on the grooved surface, and vapor condensation on condensers with grooved heat pipes occurs mainly on the fins where the liquid layer is thinner.

The proposed model is based on the following assumptions:

a) vapor condensation occurs on the fins;

b) the liquid formed in condensation flows from the film on the fin into the groove under the action of surface tension forces; the free surface of the liquid film must have a corresponding curvature;

c) the liquid entering the groove is also carried by surface tension forces along the groove from the condenser to the evaporator;

d) the force of gravity is absent;

e) friction of the vapor on the film is neglgible.

The geometry of the groove and fin can vary. In [1] the case of a rectangular fin with rounded edges was considered. A more general case of arbitrary fin form can be analyzed.

We will consider liquid condensation on a portion of a grooved surface (Fig. 1). The liquid film thickness on the fin is very small, and the Reynolds number for the flow in the film across the fin is on the order of 10^{-2} [1]. As a consequence, the convective term may be neglected in the equation of motion. For the case of a thin film satisfying the condition $\delta \ll R$, where δ is the film thickness and R is the radius of curvature of the fin, the equation of motion has the form

$$\frac{dP_l}{ds} = \mu_l \frac{d^2U}{d\eta^2}. \quad (1)$$

The heat liberated in condensation is carried off by thermal conductivity through the film. For the local thermal flux through the film, we have

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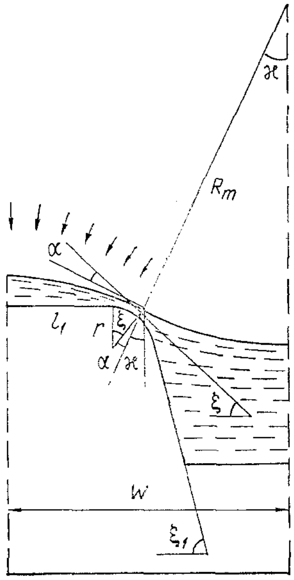

Fig. 1. Element of grooved surface. For derivation of boundary condition (11).

$$q = \lambda_l \frac{T_V - T_w}{\delta}.$$

Then the following continuity equation may be written, reflecting the law of concentration of quantity of liquid in the film:

$$\frac{d}{ds} \int_0^\delta U d\eta = \frac{\lambda_l}{L\rho_l} \frac{T_V - T_w}{\delta}. \quad (2)$$

Assuming adhesion of the liquid to the fin and absence of friction on the free film surface, from Eq. (1) the following expression can be obtained for the velocity profile:

$$U = -\frac{1}{2\mu_l} \frac{dP_l}{ds} (2\eta\delta - \eta^2). \quad (3)$$

Substituting the film velocity profile (3) into the continuity equation (2), we obtain

$$\frac{d}{ds}\left(\delta^3 \frac{dP_l}{ds}\right) = -\frac{3\mu_l \lambda_l}{L\rho_l} \frac{T_V - T_w}{\delta}. \quad (4)$$

In Eq. (4) there appears a quantity difficult to identify in experiment, T_w, the temperature of the fin wall surface. If we introduce the heat-transfer coefficient α_{ext}, which considers the thermal resistance of the heat pipe wall and the thermal resistance of external heat exchange, then in place of the temperature T_w in Eq. (4) we can introduce the temperature of the medium surrounding the thermal tube T_{ext}. As a result, Eq. (4) transforms to

$$\frac{d}{ds}\left(\delta^3 \frac{dP_l}{ds}\right) = -\frac{3\mu_l \lambda_l}{L\rho_l} \frac{T_V - T_{ext}}{\delta + \frac{\lambda_l}{\alpha_{ext}}}. \quad (5)$$

Under zero gravity conditions, the liquid in the film flows under the influence of surface tension forces. This also occurs for a horizontal surface under terrestrial conditions. Therefore, for any point of the free surface, the Laplace relationship

$$P_V - P_l = \sigma K.$$

must be satisfied. The vapor pressure P_V is assumed constant above the film surface. Then

$$\frac{dP_l}{ds} = -\sigma \frac{dK}{ds}. \quad (6)$$

Substituting Eq. (6) in Eq. (5), we obtain

$$\left(\delta+\frac{\lambda_l}{\alpha_{ext}}\right)\frac{d}{ds}\left(\delta^3\frac{dK}{ds}\right)=\frac{3\nu_l\lambda_l}{L\sigma}\ (T_V-T_{ext}). \tag{7}$$

The film free surface curvature K appearing in Eq. (7) can be expressed in terms of the fin surface curvature K_p and the film thickness δ in the following manner [1]:

$$K=K_p+\frac{\frac{d^2\delta}{ds^2}}{\left[1+\left(\frac{d\delta}{ds}\right)^2\right]^{3/2}}. \tag{8}$$

Equations (7) and (8) then represent a system of equations for determination of the two unknown functions $\delta(s)$ and $K(s)$.

From the symmetry of the surface form relative to the middle of the fin s = 0 and the equality to zero at that point of the liquid flow rate, we obtain two boundary conditions

$$\left.\frac{d\delta}{ds}\right|_{s=0}=0,\quad \left.\frac{dK}{ds}\right|_{s=0}=0. \tag{9}$$

The two remaining boundary conditions must be formulated at the other end of the film, i.e., at the point of transition of the film surface into the meniscus of the liquid in the groove. At this point the film surface curvature must be equal to the curvature of the meniscus in the groove:

$$K|_{s=l}=\frac{1}{R_m}. \tag{10}$$

This last boundary condition is obtained from the requirement that the slope of the tangents at the point $s = l$ to the film surface and to the meniscus be equal. The concrete description of this condition depends on the geometry of the fin and groove. Thus, in the case of a trapezoidal fin with planar upper surface and rounded edge, this condition is written in the following manner:

$$\frac{R_m\left(\sin\xi-\left.\frac{d\delta}{ds}\right|_{s=l}\cos\xi\right)}{\sqrt{1+\left(\frac{d\delta}{ds}\right)^2_{s=l}}}+(\delta|_{s=l}+r)\sin\xi=W-l_1, \tag{11}$$

where $\xi = (l - l_1)/r$, and is a consequence of the geometric relationship (see Fig. 1):

$$R_m\sin(\xi-\alpha)+(\delta+r)\sin\xi=W-l_1,$$

and also of the fact that $\tan\alpha = d\delta/ds$.

Boundary conditions (10), (11) must be specified at the point located at the transition from film flow to liquid flow in the groove, where Eqs. (1), (2) are still applicable. This point should be chosen farther from the middle of the fin, the larger the curvature of the meniscus in the groove.

Equations (7), (8) with boundary conditions (9)-(11) permit determination of the film thickness and curvature distribution across the fin. This information is sufficient to determine the condensation rate. In fact, the quantity of vapor condensed on a fin per unit time is equal to the liquid flow through the film section $s = l$:

$$m=\left.\int_0^\delta\rho_l U d\eta\right|_{s=l}.$$

Substituting in this expression the velocity profile in the film, Eq. (4), integrating over η, and using Eq. (6), we obtain

$$m=\frac{\sigma}{3\nu_l}\left(\delta^3\frac{dK}{ds}\right)_{s=l}. \tag{12}$$

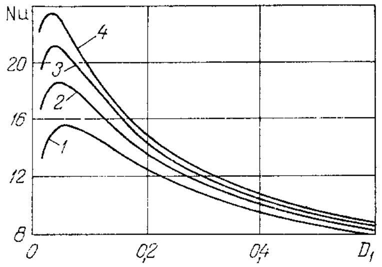

Fig. 2

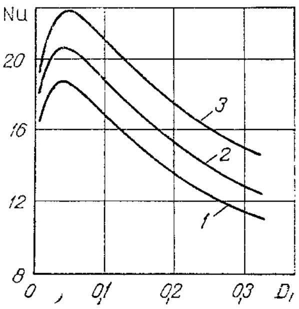

Fig. 3

Fig. 2. Number Nu vs dimensionless meniscus curvature in groove D_1, $D_3 = 0.01536$; $r/l_1 = 0.1$; $W/l_1 = 2.22$ for various D_2: 1-4) $D_2 = 30, 40, 50, 60$, respectively.

Fig. 3. Variation of function $Nu(D_1)$ with parameter D_3: $D_2 = 40$; $r/l_1 = 0.1$; $W/l_1 = 2.22$; 1-3) D_3 equal to 0.0153, 0.01, 0.005, respectively.

With the aid of Eq. (12), it is simple to obtain an expression for the heat-transfer coefficient α_c for condensation on a grooved surface. In a unit time period upon a fin of unit length there condenses a mass of liquid m (calculated for the half-sum of the fin and groove width W). An amount of heat equal to mL is then liberated. Then the mean thermal flux density is equal to q = mL/W and for the heat-transfer coefficient we have

$$\alpha_c = \frac{mL}{W(T_V - T_{ext})} = \frac{\sigma L}{3\nu_l W(T_V - T_{ext})} \left(\delta^3 \frac{dK}{ds}\right)_{s=l}. \tag{13}$$

We will transform to dimensionless variables in the problem of Eqs. (7)-(11). As the scale for the transverse coordinate s, we choose l_1, the half-width of the flat portion of the fin, and for the scale for film thickness measurement, we choose the quantity

$$\gamma = \sqrt[4]{\frac{3\nu_l \lambda_l l_1^3}{L\sigma} (T_V - T_{ext})}\,. \tag{14}$$

If we now transform to the variables

$$s' = \frac{s}{l_1}, \quad \delta' = \frac{\delta}{\gamma}, \quad K' = Kl_1, \tag{15}$$

then Eqs. (7), (8) and the boundary conditions appear as (primes omitted)

$$\frac{d^2\delta}{ds^2} = \left[1 + \frac{1}{D_2^2}\left(\frac{d\delta}{ds}\right)^2\right]^{3/2} D_2(K - K_p), \quad (\delta + D_2D_3)\frac{d}{ds}\left(\delta^3 \frac{dK}{ds}\right) = 1, \tag{16}$$

$$\left.\frac{d\delta}{ds}\right|_{s=0} = 0, \quad \left.\frac{dK}{ds}\right|_{s=0} = 0, \quad K|_{s=l/l_1} = D_1,$$

$$\frac{\sin\xi - \frac{1}{D_2}\left.\frac{d\delta}{ds}\right|_{s=l/l_1}\cos\xi}{\sqrt{1 + \frac{1}{D_2^2}\left(\frac{d\delta}{ds}\right)^2_{s=l/l_1}}} + \left(\delta|_{s=l/l_1} + D_2\frac{r}{l_1}\right)\frac{D_1}{D_2}\sin\xi = D_1\left(\frac{W}{l_1} - 1\right), \tag{17}$$

where the following notation is introduced for dimensionless combinations:

$$D_1 = \frac{l_1}{R_m}, \quad D_2 = \frac{l_1}{\gamma}, \quad D_3 = \frac{\lambda_l}{\alpha_{ext} l_1}. \tag{18}$$

The parameter D_1 specifies the dimensionless curvature of the meniscus in the groove; D_2 is the ratio of the length of the flat fin section to the film thickness scale γ, and, since it contains the temperature head $T_V - T_{ext}$, it defines the intensity of condensation; D_3 characterizes the ratio of the external thermal resistance to the thermal resistance of the film.

The expression for the heat-transfer coefficient (13) in the new dimensionless variables takes on the form

$$\alpha_c = \frac{\lambda_l}{W} D_2 \left(\delta^3 \frac{dK}{ds}\right)_{s=l/l_1}. \tag{19}$$

If we define the Nusselt number by the approximation $Nu = \alpha_c W/\lambda_l$, then from Eq. (19) we obtain

$$Nu = D_2 \left(\delta^3 \frac{dK}{ds}\right)_{s=l/l_1}. \tag{20}$$

For fins and grooves of a given geometry, the right side of Eq. (20), as is evident from Eqs. (16), (17), depends on three dimensionless parameters D_1, D_2, D_3. Thus, for heat transfer in condensation on a grooved surface of given geometry, we obtain the following generalized function:

$$Nu = F(D_1, D_2, D_3). \tag{21}$$

The form of the function $F(D_1, D_2, D_3)$ can be made clear by solving the problem of Eqs. (16), (17), the solution being performed numerically. System (16) was written in the form of a system of four first-order equations and solved by Newtonian iteration. At each iteration the matrix drive method was employed.

The point $s = l/l_1$, at which the boundary conditions are specified, was chosen at some distance Δs from the point of intersection of the fin surface with the straight line joining the centers of curvature of the meniscus of the liquid in the channel and the rounded edge of the fin. Variation of Δs proved to have a very slight effect on the results.

Calculations show that for a given heat-exchange agent and fixed groove and fin geometry, the most important factor affecting heat exchange is the curvature of the meniscus in the channel (parameter D_1). The characteristic form of the dependence of Nu on meniscus curvature is shown in Fig. 2. With the exception of the small segment corresponding to low meniscus curvature in the channel, the number Nu decreases monotonically with growth in meniscus curvature. In the region of very small curvatures, the function $Nu(D_1)$ is increasing. Such Nu behavior is caused by the form of the film created on the fin. With very small meniscus curvature in the groove the film surface is close to planar, so that the liquid drains from the fin to the groove poorly under the action of surface tension. With increase in curvature, the draining of liquid is improved, and the film becomes thinner on the average, leading to an increase in Nu. With further increase in D_1 the film thickness in the middle part of the fin increases while it decreases beyond the edge, hindering liquid drainage, so Nu falls.

Figure 2 also shows the variation of the function $Nu(D_1)$ with change in D_2. With increase in D_2, which is the ratio of the length of the plane fin segment to the characteristic film thickness dimension γ, the number Nu increases.

Variation of the function $Nu(D_1)$ with D_3 is shown in Fig. 3. With increase in D_3 Nu decreases.

The results obtained indicate that by appropriate choice of grooved condensation surface parameters, the heat-transfer characteristics may be improved.

NOTATION

P, pressure; U, velocity; S, η, coordinates across fin and normal to fin surface; μ, dynamic viscosity; T, temperature, λ, thermal conductivity coefficient; ρ, density, L, latent heat of evaporation; α, heat-transfer coefficient (angle in Fig. 1); σ, surface tension coefficient; ν, kinematic viscosity; K, curvature; R_m, radius of curvature of meniscus in groove; r, radius of curvature of fin; W, half sum of groove and fin width; l_1, half width of plane segment of fin; l, half length of film on fin. Indices: l, liquid; v, vapor; ext, external.

LITERATURE CITED

1. Y. Kamotani, "Analysis of axially grooved heat pipe condensers," AIAA Paper, No. 76-147 (1976).